An executive summary of the final report of work done on the Minor Research Project of DENZIL SALDANHA entitled

“APPLICATION OF GRAPH COLORING AND LABELING IN CRYPTOGRAPHY”

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Graph coloring is one of the most important concepts in graph theory and is used in many real time applications .Various coloring methods are available and can be used on requirement basis. The proper coloring of a graph is the coloring of the vertices and edges with minimal number of colors such that no two vertices should have the same color. The minimum number of colors is called as the chromatic number and the graph is called properly colored graph.

Graph structure can be used to study the various concepts of Navigation in space. A work place can be denoted as node in the graph, and edges denote the connections between the places. The classical problem of keeping minimum number of machines (or Robots or Broadcast stations) at certain nodes to trace each and every node uniquely, can be solved by using networks where places are interconnected in which, the navigating agent moves from one node to another in the network. It helps the society to share information among nodes securely using cryptography and graph coloring where security is maintained. However there is neither the concept of direction nor that of visibility. Instead we shall assume that a Robot navigating on a Graph can sense the distance to a set of landmarks.

For safeguard analysis of a facility, such as a fire protection, study of nuclear power plants, the facility can be modeled by a graph or network. For such applications, a vertex can represent a room, hallway, stairwell, courtyard, and etc. Each edge can connect two areas that are either physically adjacent or within sight or sound of each other. It is necessary to determine a collection of vertices at which to place detection device so that if there is an object at any vertex in the graph, it can be detected, and its position uniquely identified.

In order to detect an object, which might be at any vertex in V (G), it is necessary to have a dominating set. The additional problem of uniquely identifying the location of the stations requires a metro domination feature.

A dominating set which is also a resolving set is called a metro dominating set or in short an $MD - set$. An $MD - set$ of G is called a minimal metro dominating set of G if no proper subset of D is a metro dominating set of G. the cardinality of a minimal $MD - set$ of minimum cardinality is called the lower metro domination number or simply a metro domination number of and is denoted by $\gamma_\beta(G)$.

A metro dominating set D of a graph $G(V, E)$ is called a unique metro dominating set (in short an UMD-set) if $|N(v) \cap D| = 1$,for each vertex $v \in V - D$. A $UMDset$ of G is called a minimal unique metro domination number of G if none of its proper subsets is an$UMD - set$ for G. The minimum cardinality of a minimal $UMDset$ of a graph G is called Unique metro domination number or UniMetro domination number and is denoted by $\gamma_{\mu\beta}$(G).

It is studied that $\gamma(P_2) = 1, \gamma(P_3) = 1, \dots \dots \gamma(P_n) = 2 \; for \; n = 4,5,6$

In general the domination number of path P_n is $\gamma(P_n) = \left\lfloor \frac{n+2}{3} \right\rfloor$

Metro domination number of P_n is $\gamma_\beta(P_n) = \left\lfloor \frac{n}{3} \right\rfloor$

The study in paths $P_n$revealed the result, $\gamma_{\mu\beta}$ (P_1)=1,$\gamma_{\mu\beta}$ (P_2)==1, $\gamma_{\mu\beta}$ (P_3)=2

$$\gamma_{\mu\beta}(P_n) = \begin{cases} 1 & if \; n = 1,2 \\ 2 & if \; n = 3 \\ \left\lfloor \frac{n+2}{3} \right\rfloor & otherwise \end{cases}$$

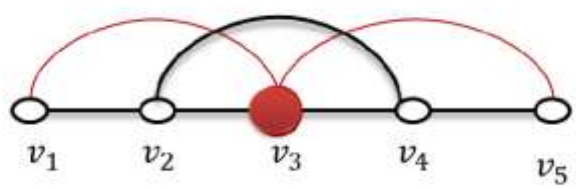
Here the number of nodes representing workplace or stations is not minimal even though unique metro dominating nodes are obtained.

Define a path called ${P_n}^2$

Let $G(V, E)$ be a connected graph ,second power of G denoted by G^2 whose vertex is same as that of G and two vertices (u, v) in G^2 are adjacent if and only if $d(u, v) \leq 2$ in G.

The domination number of ${P_n}^2$ were studied and found that

$$\gamma({P_n}^2) = \begin{cases} 1 & if\ n = 3,4 \\ \left\lceil \frac{n}{5} \right\rceil & if\ n \geq 5 \end{cases}$$



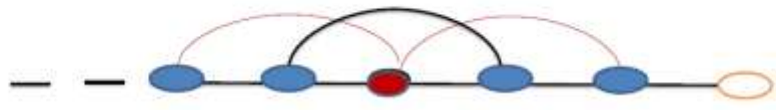
Here the dominating set $D = \{v_3\}$ and domination number is $\gamma({P_5}^2) = 1$

Here one broadcast station is sufficient to transmit messages to cities. Here we minimize the number of broadcast stations but the messages passed through these stations are not secured ,the information is shared among other cities.

So the study of Metro domination of ${P_n}^2$ is done :

It is found that : For any positive integer n, $\gamma({P_n}^2) \geq \left\lceil \frac{n}{5} \right\rceil$

1. If $n = 5k$, then $\gamma({P_n}^2) = k$
2. When $n = 5k + 1$, $\gamma({P_n}^2) = k + 1$



3. When $n = 5k + 2$, $\gamma({P_n}^2) = \left\lceil \frac{n}{5} \right\rceil$
4. When $n = 5k + 3$, $\gamma({P_n}^2) = \left\lceil \frac{n}{5} \right\rceil$
5. When $n = 5k + 4$, $\gamma({P_n}^2) = \left\lceil \frac{n}{5} \right\rceil$

Summarizing these , we get then the theorem

Theorem 1: For $n \geq 3$, $\gamma({P_n}^2) = \left\lceil \frac{n}{5} \right\rceil$

Theorem 2: If $n \geq 13$ then $\gamma_\beta({P_n}^2) = \left\lceil \frac{n}{5} \right\rceil$

Therefore we obtain the result which states that

For any integer $n \geq 3$,$\gamma_\beta({P_n}^2) = \begin{cases} 2 & 3 \leq n \leq 7 \\ 3 & 8 \leq n \leq 10 \\ \left\lceil \frac{n}{5} \right\rceil & n \geq 11 \end{cases}$